Problem 1. (10 Points)

a. (5) Consider the proposition: "If the moon is green then the sun is blue". Is it True or false? Why ?

b. (5) Use a truth table to decide whether $p \rightarrow (q \rightarrow r)$ is equivalent to $(p \wedge q) \rightarrow r$ or not.

p	q	r	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Problem 2. (10 Points)

Consider the following argument:

" If the program is efficient, then it executes quickly. Either the program is efficient or it has a bug. However, the program does not execute quickly.Therefore it has a bug."

a. (4) Give a symbolic representation of the argument using the symbols p, q, and b where p is "the program is efficient", q is "the program executes quickly", and b is "the program has a bug".

b. (6) Is the argument above valid or not? Justify your answer.

Problem 3. (15 Points)
In this problem, suppose that the domain of x is all computer science courses, and consider the following predicates:

I(x) : "x is interesting"
$U(x)$: "x is useful"
$H(x,y)$: "x is harder than y"
$M(x,y)$: "x has more students than y"

Write the following statements in predicate logic:

a. (3) All interesting computer science courses are useful

b. (3) There are some useful computer science courses that are not interesting

c. (4) Every interesting computer science course has more students than any non interesting computer science course

d. (5) Write the following predicate logic statement in everyday English. Don't just give a word-for-word translation; your sentence should make sense.

$$\exists x \; (\; I(x) \land \forall y \; (\; H(x,y) \rightarrow M(x,y) \;).$$

Problem 4 (15 Points)

Consider the logical operator NAND where the proposition p NAND q is true when either p is false, or q is false, or both are false. It is false only when both p and q are true. The NAND operator is denoted by |. (p NAND q is denoted by p | q)

a. (3) Give the truth table for the NAND operator.

b. (3) Show that $p \mid q$ is logically equivalent to $\neg(p \wedge q)$.

c. (4) Show that $\neg p$ is equivalent to $p \mid p$.

d. (4) Show that $p \wedge q$ is equivalent to $(p \mid q) \mid (p \mid q)$

Problem 5 (15 Points)

Recall that a real number x is **rational** if it can be written as the quotient of two integers; i.e. $x = m/n$, where m and n are integers; Otherwise, x is said to be **irrational**. For each of the following, prove or disprove. In each case tell what kind of a proof did you give (direct, indirect, etc…)

a. (5) The sum of two rational numbers is rational.

b. (5) The sum of two irrational numbers is irrational.

c. (5) The sum of a rational number and an irrational number is irrational.

Problem 6 (10 Points)

a. (5) Show that the union of two countably infinite sets is countably infinite.

b. (5) Using (a) or otherwise, show that the difference of an uncountable set and a countable set is uncountable.

Problem 7 (15 Points)

The symmetric difference of two sets A and B, denoted by $A \oplus B$ is the set of elements that are in one of the sets but not in both; i.e. $A \oplus B = (A - B) \cup (B - A)$.

a. (3) Draw a Venn Diagram that represents $A \oplus B$.

b. (6) Show that $A \oplus B = (A \cup B) – (A \cap B)$ Note: Venn Diagrams do not give proofs!!!

c. (6) Show that if $A \oplus B = \varnothing$ then $A=B$

Problem 8 (10 Points)

Consider the function: $f : \mathbf{Z} \to \mathbf{Z}$ where $f(x) = \begin{cases} x\text{-}4 & \text{if } x \geq 3 \\ x + 2 & \text{if } x \leq 2 \end{cases}$

where **Z** denotes the set of integers.

a. (5) Is *f* one-to-one? Why?

b. (5) Is *f* onto? Why ?

Problem 9 (10 Points).

Suppose $g : A \to B$ and $f : B \to C$, where $f \circ g$ is one-to-one. Prove or disprove:

a. (5) g is one-to-one.

b. (5) f is one-to-one.

Problem 10 (10 Points)

For each of the following, find a formula that generates the sequence $a_1, a_2, a_3 \ldots$.

a. (5) 17, 22, 27, 32, 37, …

$a_n =$

b. (5) 1, 3, 1, 3, 1, 3, …

$a_n =$

Problem 11 (10 Points)

a. (5) Show that the set of natural numbers divisible by 5 but not by 3 is countable (spell out what is the listing recipe!!)

b. (5) Suppose that x is a real number, and that n is an integer. Show that $\lfloor x+n \rfloor = \lfloor x \rfloor + n$.